Chapter 36: Diffraction

Group Members:

1. A monochromatic light of wavelength 585 *nm* falls on a slit of 0.0650 *mm* wide. A diffraction pattern is viewed on a screen *L*=2.00 *m* away.

   a. What is the angular location θ_1 of the first dark fringe on either side of the central maximum?

   $a\sin\theta = m\lambda$ or $\sin\theta = \dfrac{m\lambda}{a} = 0.00900\,(m{=}1)$

   If we keep θ in rad, $\theta \approx \sin\theta$ for $\theta \ll 1$.

   So, $\theta_1 \approx \sin\theta_1 = \dfrac{\lambda}{a} rad = \left(\dfrac{585\times10^{-9}m}{6.50\times10^{-5}m}\right) rad = 0.00900 rad$

   b. What is the linear distance *y* (in *mm*) on the wall of this dark fringe with respect to the central maximum?

   From geometry, $\tan\theta = \dfrac{y}{L}$. Again, for $\theta \ll 1$, $\theta \approx \sin\theta \approx \tan\theta$,

   So, $\dfrac{y}{L} = 0.00900$ and $y = (0.00900)2.00m = 0.180m = 18.0cm$

   c. How wide (in *mm*) is the central maximum of this diffraction pattern on the wall? [The width of the central max is defined as the separation between the first dark fringes on either side of the central bright fringe.]

   Width of central max $= 2y = 0.360m = 36.0cm$

d. How many dark fringes are visible along the wall? [θ cannot be larger than 90° to the left and to the right with respect to the center line between the slit and the wall.]

- Setting $\theta = \frac{\pi}{2}$, we can solve for the smallest m_s such that $\sin\left(\frac{\pi}{2}\right) = 1 < \frac{m_s \lambda}{a}$.

- $m_s = \text{int}\left(\frac{a}{\lambda}\right) = \text{int}(111.1111) = 111$. This give the largest order minimum before $\theta > \frac{\pi}{2}$.

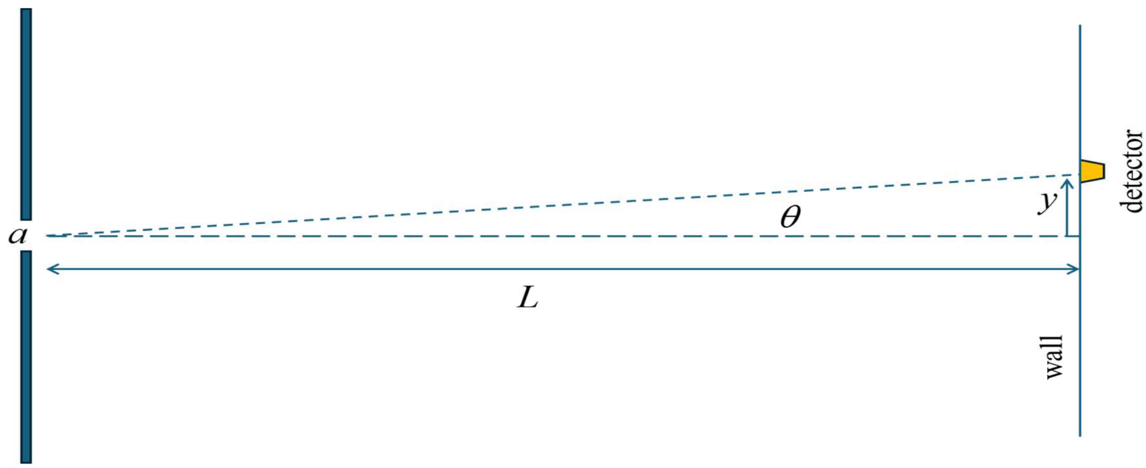
- So, we have 111 minima on the left and symmetrically 111 minima on the right. That gives a total of 222 minima in the diffraction pattern.

e. An experimenter put an intensity detector on the wall $y = 5.00cm$ above the middle of the central bright fringe. If the intensity at the central bright fringe is I_o. What is the relative intensity I/I_o that the detector will read at that location?

For a single slit diffraction pattern, the relative intensity as a function of θ is given by,

$$I(\theta) = I_o \left(\frac{\sin(\beta)}{\beta}\right)^2 \quad \text{where } \beta = \frac{\pi a \sin\theta}{\lambda}$$

Now, let calculate the angular location of the detector,



$$\tan\theta = \frac{y}{L} = \frac{5.00\times10^{-2}m}{2.00m} = 2.50\times10^{-2}$$

$$\theta = 0.0250rad$$

Now, putting this in the intensity equations, we have,

$$\beta = \frac{\pi a(0.0250)}{\lambda} = \frac{\pi\left(0.065\times10^{-3}m\right)(0.0250)}{585\times10^{-9}m} = 8.72666$$

For calculating above, we used the approximation of $\sin\theta \approx \theta$ for θ small.

Then,

$$I(\theta) = I_o\left(\frac{\sin(8.7266)}{8.7266}\right)^2 = 0.00543I_o$$

2. The diameter of a human pupil (a circular aperture) can vary between 2-4 *mm* (in bright light).

   a. Assuming the resolving power of the human eye is diffraction limited (that is the ideal situation assuming the eye acts like a perfect lens without any biology related limitations), what is the smallest angular separation $\theta_{\min}$ that the eye can discern according to the Rayleigh's criterion? Assume the light to be a reddish light with $\lambda = 675nm$ and assume that the diameter of your pupil is $D = 3.00mm$.

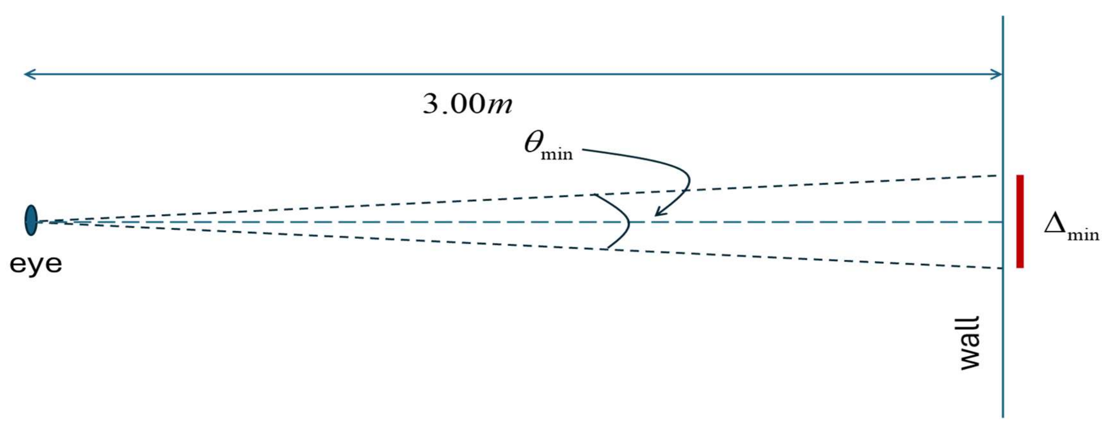
   The smallest angular separation $\theta_{\min}$ for a circular aperture according to Rayleigh's criterion is given by,

$$\sin\theta_{\min} = 1.22\frac{\lambda}{D} = 1.22\left(\frac{675\times10^{-9}m}{3.00\times10^{-3}m}\right) = 2.745\times10^{-4}$$

   Since $\lambda/D \ll 1$, we can use the approximation $\theta \approx \sin\theta \approx \tan\theta$ as long as θ is **in radian**. So, $\theta_{\min} = 2.75\times10^{-4}$

   b. Two small dots are separated by 0.750mm on a wall which is at a distance $L = 3.00m$ away. Will your eyes be able to see them as separate dots?

   With $\theta_{\min}$ above, we can calculate the minimum separation $\Delta_{\min}$ (red bar) that the two dots must be separated for them to be discerned according to Rayleigh's criterion.



Using the right triangle in the above diagram, we have

$$\tan\left(\frac{\theta_{\min}}{2}\right) \approx \frac{\theta_{\min}}{2} = \frac{half\ width}{L} = \frac{\Delta_{\min}/2}{3.00m} \qquad or \qquad \theta_{\min} = \frac{\Delta_{\min}}{3.00m}$$

So, $\Delta_{\min} = (3.00m)\theta_{\min} = 8.24\times10^{-4}m = 0.824mm$

Since $0.750mm < \Delta_{\min}$, the two dots cannot be discerned by the eye using the reddish light.

c. Now, if you illuminate the two dots with a blue light with a shorter wavelength $\lambda = 425nm$. Will your eyes be able to discern the two dots as separate dots?

The smallest angular separation $\theta_{\min}$ using a blueish light $\lambda = 425nm$ will be smaller,

$$\sin\theta_{\min} = 1.22\frac{\lambda}{D} = 1.22\left(\frac{425\times10^{-9}m}{3.00\times10^{-3}m}\right) = 1.728\times10^{-4}$$

And,

$$\Delta_{\min} = (3.00m)\theta_{\min} = 5.19\times10^{-4}m = 5.19mm$$

Since $0.750mm > \Delta_{\min}$, so using a blue light with a smaller $\theta_{\min}$, the eyes will have a better chance in discerning the two dots.

d. A tiger has a much large diameter $D = 30.0mm$ for its pupil. What is the smallest angular separation that the eye can discern according to the Rayleigh's criterion? For this calculation, assume a mid-range wavelength $\lambda = 550nm$ in the visual light spectrum.

Similar to the other calculations,

$$\theta_{\min} \approx \sin\theta_{\min} = 1.22\frac{\lambda}{D} = 1.22\left(\frac{550\times10^{-9}m}{3.00\times10^{-2}m}\right) = 2.24\times10^{-5}$$

This minimum angular separation is much smaller the one for human. That is why tigers have sharper vision than human.